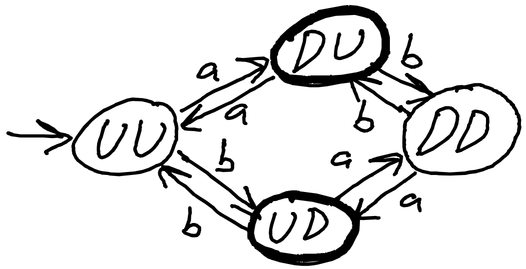
# Lecture 8 : DFA minimisation example 1

Example DFA from Lec. 1 : a light controlled by two switches. Both switches are initially in the up position and the light is off. Either switch can toggle the light on/off.




$L = \{ w \in \{a,b\}^* : \text{len}(w) \text{ is odd} \}$

Can we find the suffix equivalence classes?

Consider this partition of the set of states:

$\{\{UU, DD\}, \{DU, UD\}\}$

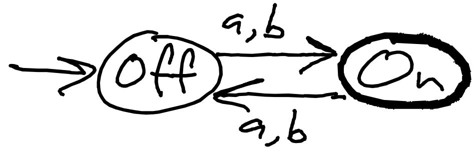
Are there distinguishing extensions for words leading to UU and DD?

What about DU and UD?

Or $\{UU, DD\}$ and $\{DU, UD\}$?

How many suffix-equivalence classes do we have?

The minimal automata for this system:




The Myhill-Nerode theorem would construct state Off as $[\epsilon]_{\sim_{suffix}}$ and On as $[a]_{\sim_{suffix}}$

where $[\epsilon]_{\sim_{suffix}} = \{ w \in \{a,b\}^* : \text{len}(w) \text{ is even} \}$

and $[a]_{\sim_{suffix}} = \{ w \in \{a,b\}^* : \text{len}(w) \text{ is odd} \}$

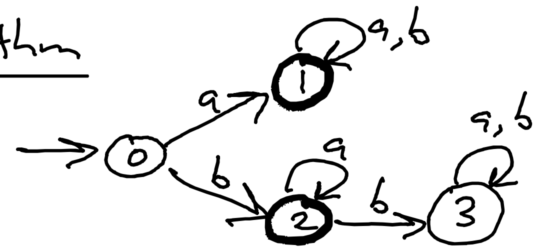
We can also see that

$[\epsilon]_{\sim_{suffix}} = \{\text{words leaving orig. DFA in UU}\} \cup \{\text{words leaving orig. DFA in DD}\}$

$[a]_{\sim_{suffix}} = \{\text{words leaving orig. DFA in DU}\} \cup \{\text{words leaving orig. DFA in UD}\}$

## A DFA minimisation algorithm

Consider the DFA :



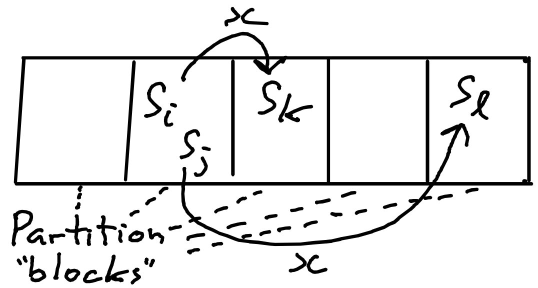

Initial coarse partition of states :

1) Accepting : $\{1,2\}$

2) Non-accepting : $\{0,3\}$

Is the partition "stable" under the transition function? It is unstable if this occurs:

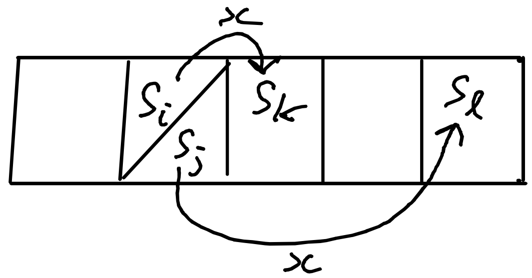
Partition P =




If P is stable, we're done.

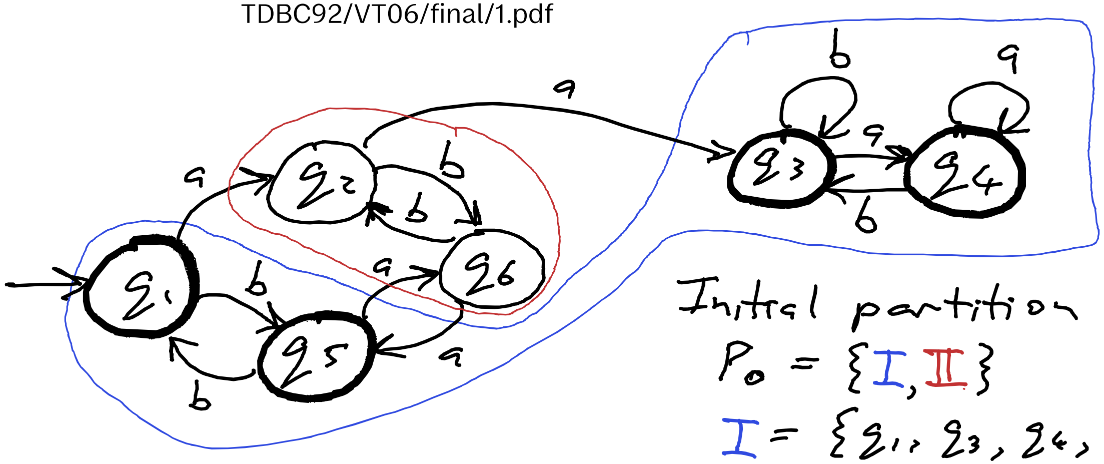
If not, split the part containing $s_i$ and $s_j$ into multiple parts that <u>each</u> agree on the destination block under each input character.

P =



# Example of DFA minimisation

https://web.archive.org/web/20150621004621/http://www8.cs.umu.se/kurser/TDBC92/VT06/final/1.pdf




Initial partition:

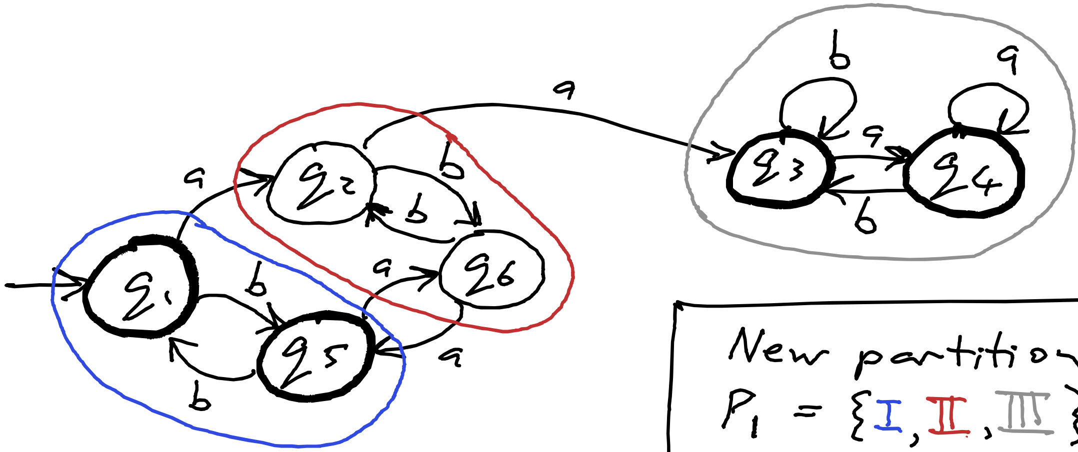
$P_0 = \{\mathrm{I}, \mathrm{II}\}$

$\mathrm{I} = \{q_1, q_3, q_4, q_5\}$

$\mathrm{II} = \{q_2, q_6\}$

$\mathrm{I} \xrightarrow{a} \{\mathrm{I}, \mathrm{II}\}$ $\quad \mathrm{I} \xrightarrow{b} \{\mathrm{I}\}$

$\mathrm{II} \xrightarrow{a} \{\mathrm{I}\}$ $\quad \mathrm{II} \xrightarrow{b} \{\mathrm{II}\}$




$\mathrm{I} \xrightarrow{a} ?$ $\qquad$ $\mathrm{I} \xrightarrow{b} ?$

$\mathrm{II} \xrightarrow{a} ?$ $\qquad$ $\mathrm{II} \xrightarrow{b} ?$

$\mathrm{III} \xrightarrow{a} ?$ $\qquad$ $\mathrm{III} \xrightarrow{b} ?$

New partition:

$P_1 = \{\mathrm{I}, \mathrm{II}, \mathrm{III}\}$

$\mathrm{I} = \{q_1, q_5\}$

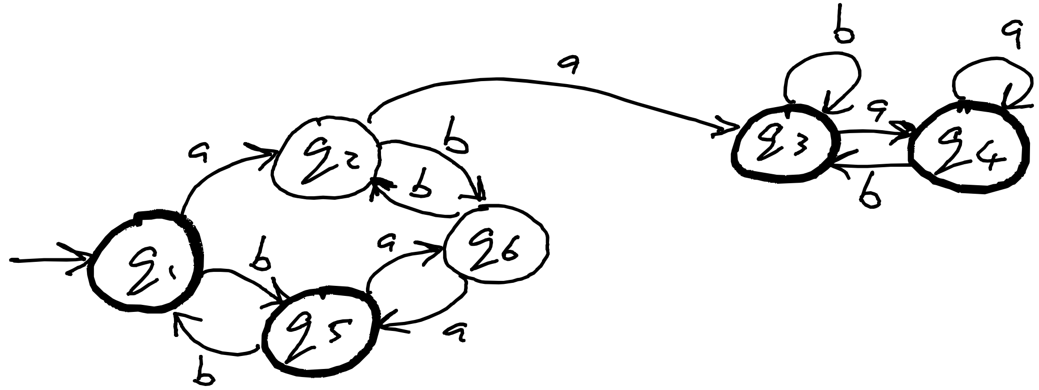
$\mathrm{II} = \{q_2, q_6\}$

$\mathrm{III} = \{q_3, q_4\}$

What is the next split?

Exercise: complete this process
(See URL on previous slide if you get stuck)

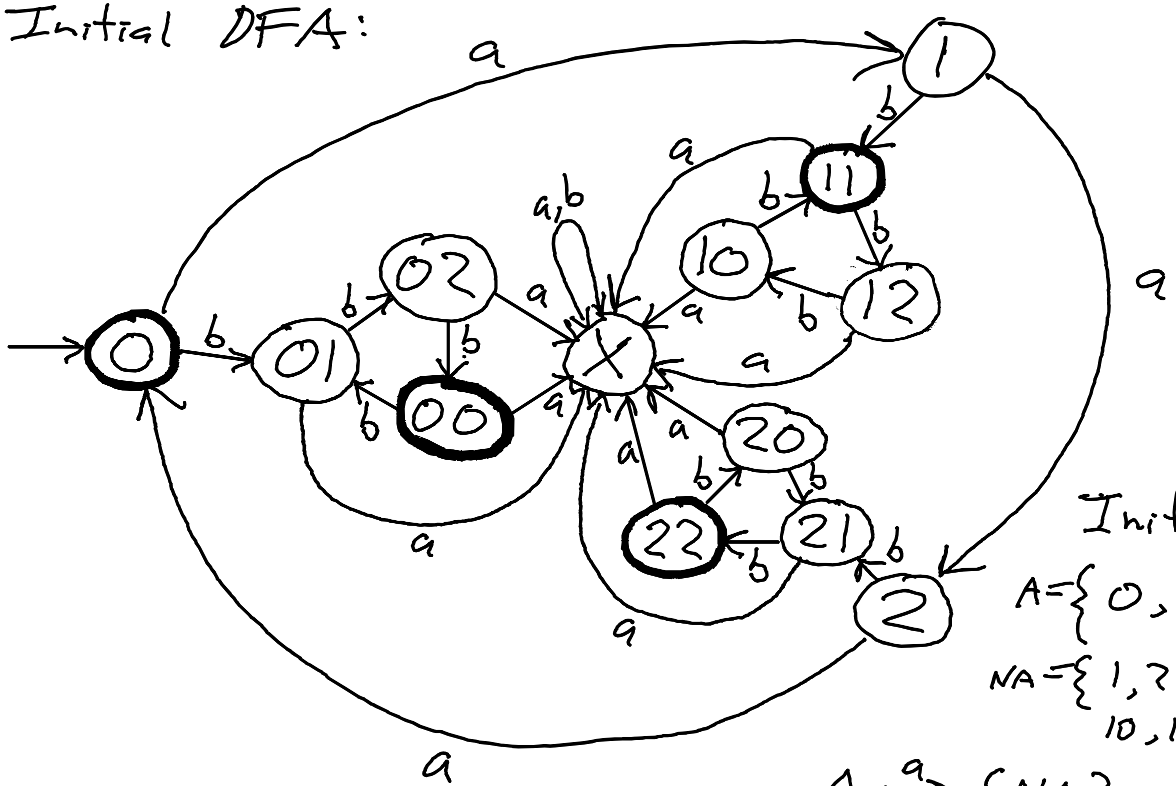
Original diagram for copying & pasting



# Another DFA minimisation example

$L = \{a^n b^k : k-n \text{ is a multiple of } 3\}$

Initial DFA:




Initial partition

$A = \{0, 00, 11, 22\}$

$NA = \{1, 2, 01, 02, 10, 12, 20, 21, X\}$

$A \xrightarrow{a} \{NA\}$ $A \xrightarrow{b} \{NA\}$

$NA \xrightarrow{a} \{NA, A\}$

$NA \xrightarrow{b} \{NA, A\}$

Needs splitting

Partition transitions for NA:

| State | a | b |
|---|---|---|
| 1 | NA | A |
| 2 | A | NA |
| 01 | NA | NA |
| 02 | NA | A |
| 10 | NA | A |
| 12 | NA | NA |
| 20 | NA | NA |
| 21 | NA | A |
| X | NA | NA |

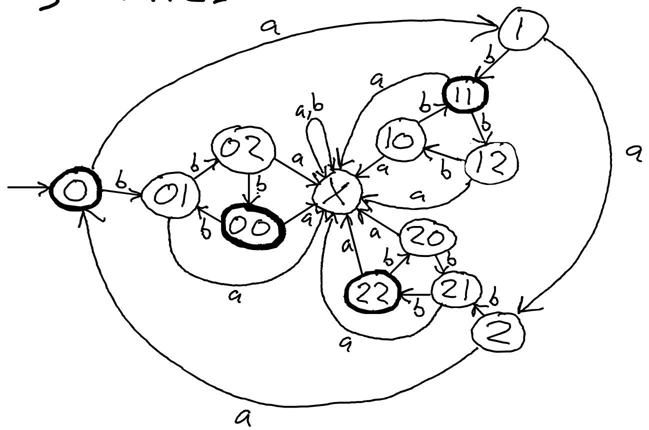
{1, 02, 10, 21} : B

{2} : C

{01, 12, 20, X} : D

still the accepting states

New partition : A, B, C, D




<u>Next iteration</u>

$A \xrightarrow{a} \{B, D\}$ $\quad A \xrightarrow{b} \{D\}$

$B \xrightarrow{a} \{C, D\}$ $\quad B \xrightarrow{b} \{A\}$

Need to split A and B!

A

| State | a | b |
|---|---|---|
| 0 | B | D |
| 00 | D | D |
| 11 | D | D |
| 22 | D | D |

{0} : E

{00, 11, 22} : F

B

| State | a | b |
|---|---|---|
| 1 | C | A |
| 02 | D | A |
| 10 | D | A |
| 21 | D | A |

{1} : G

{02, 10, 21} : H

The partition is now:

C, E & G can't be split

$D \xrightarrow{a} \{D\}$ $\quad D \xrightarrow{b} \{D, H\}$

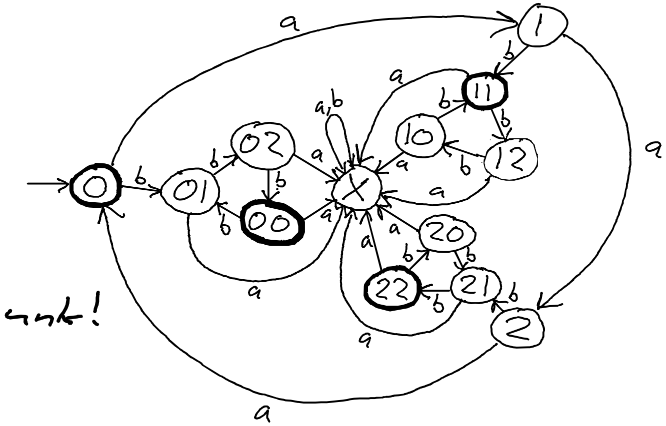
$F \xrightarrow{a} \{D\}$ $\quad F \xrightarrow{b} \{D\}$

$H \xrightarrow{a} \{D\}$

| | |
|---|---|
| {2} | C |
| {01, 12, 20, X} | D |
| {0} | E |
| {00, 11, 22} | F |
| {1} | G |
| {02, 10, 21} | H |

Need to split D

| | State | a | b | |
|---|---|---|---|---|
| D | 01 | D | H | {01, 12, 20} : I |
| | 12 | D | H | |
| | 20 | D | H | |
| | X | D | D | {X} : J for junk! |




New partition:

| | |
|---|---|
| {2} : C | {0} : E |
| {00, 11, 22} : F | {1} : G |
| {02, 10, 21} : H | |
| {01, 12, 20} : I | {X} : J |

C, E, G & J can't be split

$F \xrightarrow{a} \{J\}$ $\quad F \xrightarrow{b} \{I\}$

$H \xrightarrow{a} \{J\}$ $\quad H \xrightarrow{b} \{F\}$

$I \xrightarrow{a} \{J\}$ $\quad I \xrightarrow{b} \{H\}$

The partition is stable, so we are done!

The minimal DFA: