COSC341 TUTORIAL 2

The theme of this tutorial is to explore the definition of strings a bit, and to understand the relationships between regular grammars and DFAs.

Recall that a string of length n over an alphabet Σ is actually a function from the set $\{0, 1, 2, \ldots, n-1\}$ (which I'll denote $[n]$) to Σ and that the set of all strings (of any length, including the empty string) over Σ is denoted Σ^*. The notation for the length of a given string w is $|w|$.

1. If Σ has k elements how many different strings of length n are there over Σ?
2. If $w \in \Sigma^*$ and $m, i + j \leq |w|$ how are:
   - the prefix of w of length m,
   - the suffix of w of length m, and
   - the substring of w starting at position i of length j

   defined?
3. If $w, v \in \Sigma^*$ how are:
   - the result of prepending $a \in \Sigma$ to w,
   - the result of appending $a \in \Sigma$ to w, and
   - the result of concatenating w and v

   defined?
4. Determine a regular grammar for the language of all strings containing an even number of as (over $\Sigma = \{a, b\}$).
5. Let $\mathbf{A}$ and $\mathbf{B}$ be DFAs over the same alphabet, Σ. Can you describe DFAs that accept:
   - The complement of the language $L(\mathbf{A})$, i.e., the set of all strings <u>not</u> belonging to $L(\mathbf{A})$,
   - The intersection of $L(\mathbf{A})$ and $L(\mathbf{B})$, i.e., the set of all strings belonging to <u>both of</u> $L(\mathbf{A})$ and $L(\mathbf{B})$, and
   - The union of $L(\mathbf{A})$ and $L(\mathbf{B})$, i.e., the set of all strings belonging to <u>at least one of</u> $L(\mathbf{A})$ and $L(\mathbf{B})$.
6. Repeat the previous question for the languages $L(G)$ and $L(H)$ associated to two right-regular grammars over Σ.