COSC341 TUTORIAL 6

Doing some exercises about equivalence relations in general and then the particular versions associated to languages and DFAs. As usual, $\Sigma = \{a, b\}$ unless otherwise specified.

1. Which of the following are equivalence relations on the set $\mathbb{P} = \{1, 2, 3, \dots\}$ of positive integers? For those which are, describe the equivalence classes, and determine how many there are.

   - $a \sim b$ means "a is a divisor of b".
   - $a \sim b$ means "the prime divisors of a and b are the same"
   - $a \sim b$ means "some digits (base 10) of a can be deleted to give b"
   - $a \sim b$ means "the sum of the digits (base 10) of a and b are the same"
   - $a \sim b$ means "either both a and b are less than 100, or both are greater than or equal to 100"
   - $a \sim b$ means "$a - b$ is a multiple of 3 or of 5"
   - $a \sim b$ means "$a - b$ is a multiple of 3 and of 5"

In the next lecture we will discuss two equivalence relations on words in Σ^* given a DFA $\mathbf{A}$ with alphabet Σ:

state-equivalence ($\sim_{\text{state}}$):

$w \sim_{\text{state}} v \iff$ the state reached in $\mathbf{A}$ by processing v is the same as that reached by processing w.

suffix-equivalence ($\sim_{\text{suffix}}$):

$w \sim_{\text{suffix}} v \iff$ for every word y, either <u>both</u> wy and vy are accepted by $\mathbf{A}$ or <u>neither is</u>

Suffix-equivalence holds when the sets of possible suffixes we can attach to w or v to produce words that are accepted by $\mathbf{A}$ are the same.

2. Suppose that a DFA, $\mathbf{A}$ has two different garbage states. Are words that lead to one garbage state state-equivalent to words that lead to the other? Are they suffix-equivalent?

3. We will generate some small DFAs randomly and figure out the state- and suffix- equivalence relations.

4. The Myhill-Nerode Theorem (still to be introduced in lectures) says:

   > A language is regular if and only if its suffix-equivalence relation has only finitely many equivalence classes.

   Use this theorem to show that none of the following languages are regular:

   - $L = \{a^n b^n \,:\, n \geqslant 0\}$
   - L is the set of palindromes – words that equal their own reverses, e.g., aba, $babab$, a, etc.

5. Use the Myhill-Nerode theorem to show that the following language is regular:

$$L = \{a^n b^k \,:\, 17 \text{ is a divisor of } k - n\}$$