# COSC341 TUTORIALS 9 AND 10

Exercises about building pushdown automata, context-free grammars, and checking whether or not languages are context-free. As usual, $\Sigma = \{a, b\}$ unless otherwise specified.

1. Build a PDA and describe a context-free grammar for each of the following languages:

   (a) $\mathsf{Equal}$, the set of strings having the same number of $a$'s as $b$'s in any order.

   (b) $\{a^n b^n c^m \mid n, m \geq 0\}$.

   (c) $\mathsf{BalancedParentheses}$, the set of strings over $\{(,), a, b\}$ in which the parentheses are properly balanced (and the other symbols can occur arbitrarily).

   (d) $\mathsf{PostFix}$, the set of strings over $\{a, +, -\}$ that represent legitimate expressions written in postfix notation, where $+$ is a binary operator, and $-$ a unary operator. That is, $a$ should be accepted and pushed at any time, $+$ can be applied only if there are at least two elements in the stack and reduces the stack size by 1, and $-$ can be applied only if the stack is non-empty and does not change the size of the stack.

2. Verify that if $L$ and $K$ are context-free languages, and $R$ is a regular language then all of $LK$, $L \cup K$, $L^*$ and $L \cap R$ are context-free.

3. Apply the pumping lemma and write out detailed arguments showing that the following languages are not context-free:

   (a) $\{a^n b^m a^n b^m \mid n, m \geq 0\}$.

   (b) $\{a^p \mid p \text{ is prime}\}$.

   (c) $\{a^n b^n a^n \mid n \geq 0\}$.

4. Find two context-free languages whose intersection is not context-free.

5. Show, by intersection with a suitable regular language and deriving a contradiction, that $\mathsf{Square}$, the set of all words of the form $ww$, where $w \in \{a, b\}^*$ is not context-free.

6. We saw in Question 3(c) that $\{a^n b^n a^n \mid n \geq 0\}$ is not context-free. Imagine a machine like a PDA that uses a queue rather than a stack. How could you recognise this language?